SLOPE—A MEASURE OF STEEPNESS 2.1.2 – 2.1.4

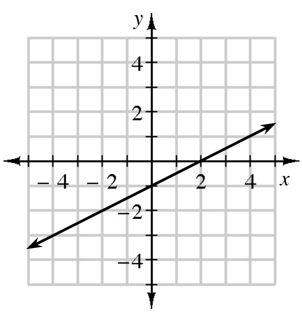
Students used the equation $y = mx + b$ to graph lines and describe patterns in previous courses. Lesson 2.1.1 is a review of writing linear equations. When the equation of a line is written in $y = mx + b$ form, the coefficient m represents the slope of the line. Slope indicates the direction of the line and its steepness. The constant term b is the y-intercept, written $(0, b)$, and indicates where the line crosses the y-axis.

For additional information about slope, see the Math Notes box in Lesson 2.1.4.

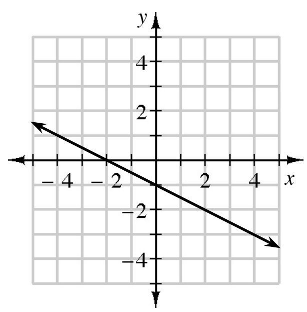
Example 1

If m is positive, the line goes upward from left to right. If m is negative, the line goes downward from left to right. If $m = 0$ then the line is horizontal. The value of b indicates the y-intercept.

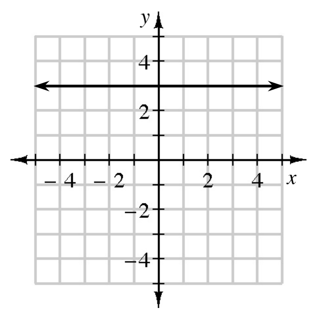
$y=\frac{1}{2}x-1$



$y=-\frac{1}{2}x-1$



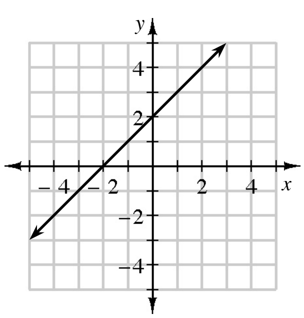
$y=0x+3$ or $y=3$



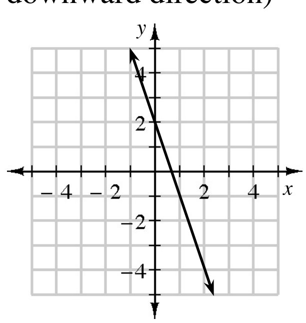
Example 2

When $m = 1$, as in $y = x$ (or $y = 1x + 0$), the line goes upward by one unit each time it goes over one unit to the right. Steeper lines have a greater m value, that is, $m > 1$ or $m < -1$. Flatter lines have an m-value that is between -1 and 1, often in the form of a fraction. All three examples below have $b = 2$ (a y-intercept of 2).

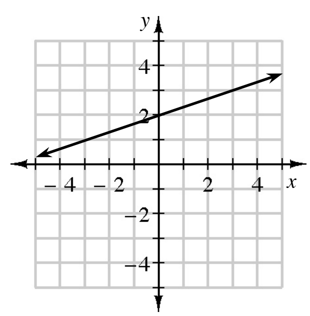
$y = x + 2$



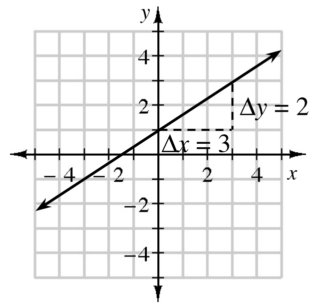
$y = -3x + 2$
(steeper and in the downward direction)



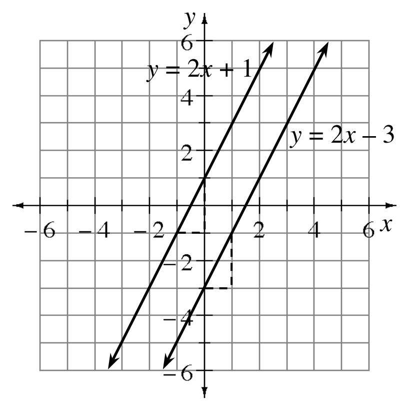
$y=\frac{1}{3}x+2$
(less steep)



Example 3



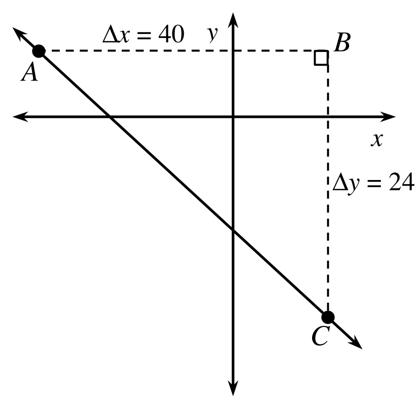
If a line is drawn on a set of axes, a *slope triangle* can be drawn between any two convenient points (usually where grid lines cross), as shown in the graph at right. Count the vertical distance (labeled Δy) and the horizontal distance (labeled Δx) on the dashed sides of the slope triangle. Write the distances in a ratio: slope $= m = \frac{\Delta y}{\Delta x} = \frac{2}{3}$. The symbol Δ means change. The order in the fraction is important: the numerator (top of the fraction) must be the vertical distance and the denominator (bottom of the fraction) must be the horizontal distance. The slope of a line is constant, so the slope ratio is the same for any two points on the line.



Parallel lines have the same steepness and direction, so they have the same slope, as shown in the graph at right.

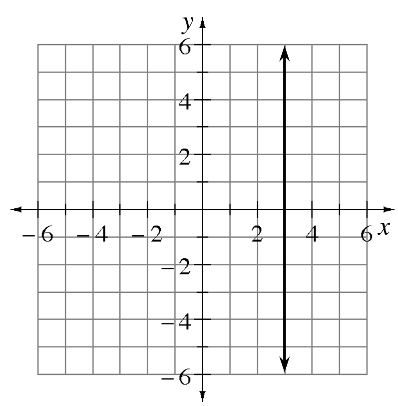
If $\Delta y = 0$, then the line is horizontal and has a slope of zero, that is, $m = 0$. If $\Delta x = 0$, then the line is vertical and its slope is undefined, so we say that it has no slope.

Example 4



When the vertical and horizontal distances are not easy to determine, you can find the slope by drawing a generic slope triangle and using it to find the lengths of the vertical Δy and horizontal (Δx) segments. The figure at right shows how to find the slope of the line that passes through the points (–21, 9) and (19, –15). First graph the points on unscaled axes by approximating where they are located, and then draw a slope triangle. Next find the distance along the vertical side by noting that it is 9 units from point B to the x-axis then 15 units from the x-axis to point C, so Δy is 24. Then find the distance from point A to the y-axis (21) and the distance from the y-axis to point B (19). Δx is 40. This slope is negative because the line goes downward from left to right, so the slope is $m = \frac{\Delta y}{\Delta x} = -\frac{24}{40} = -\frac{3}{5}$.

Example 5

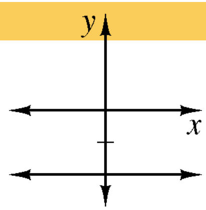


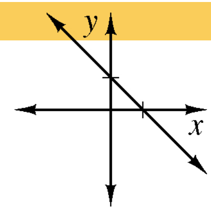
The equation of a vertical line is x = a number. For example, the graph at right shows the line $x = 3$. Every point on the line has an x-coordinate of 3.

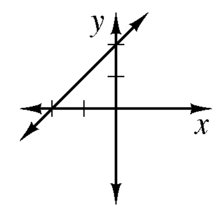
The slope of the line is $m = \frac{\Delta y}{\Delta x} = \frac{\text{any number}}{0}$ and since division by zero is not possible, the slope is undefined.

Problems

Is the slope of each line negative, positive, or zero?

1. 

2. 

3. 

Identify the slope in each equation. State whether the graph of the line is steeper or flatter than $y = x$ or $y = -x$, whether it goes up or down from left to right, or if it is horizontal or vertical.

4. $y = 3x + 2$
5. $y = -\frac{1}{2}x + 4$
6. $y = \frac{1}{3}x - 4$
7. $4x - 3 = y$
8. $y = -2 + \frac{1}{2}x$
9. $3 + 2y = 8x$
10. $y = 2$
11. $x = 5$
12. $6x + 3y = 8$

Without graphing, determine the slope of each line based on the given information.

13. $\Delta y = 27$ and $\Delta x = -8$
14. $\Delta x = 15$ and $\Delta y = 3$
15. $\Delta y = 7$ and $\Delta x = 0$
16. horizontal $\Delta = 6$ vertical $\Delta = 0$
17. between $(5, 28)$ and $(64, 12)$
18. between $(-3, 2)$ and $(5, -7)$