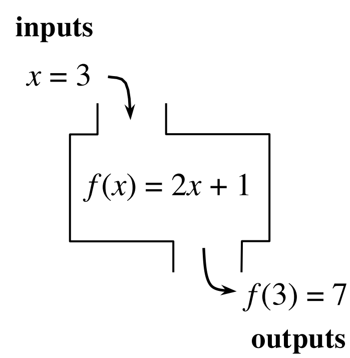
A relationship between the input values (usually x) and the output values (usually y) is called a function if for each input value, there is no more than one output value. Functions can be represented with an illustration of a "function (input–output) machine", as shown in Lesson 1.2.3 of the textbook and in the diagram in Example 1 below. Note: $f(x) = 2x + 1$ is equivalent to $y = 2x + 1$.

The set of all possible inputs is called the **domain**, while the set of all possible outputs is called the **range**.

For additional information about functions, function notation, and domain and range, see the Math Notes box in Lesson 1.2.3.

Example 1

The inputs of a function are "x"s and the outputs are "$f(x)$"s. Numbers are input into the function machine labeled f one at a time, and then the function performs the indicated operation on each input to determine its corresponding output. For example, when $x = 3$ is put into the function machine f at right, the function multiplies the 3 by 2 and then adds 1 to get the corresponding output, which is 7. The notation $f(3) = 7$ shows that the function named f connects the input 3 with the corresponding output 7. This also means the point (3, 7) lies on the graph of the function.



Example 2

a. If $f(x)=\sqrt{x-2}$ then $f(11)=?$

$f(11)=\sqrt{11-2}$
$f(11)=\sqrt{9}$
$f(11)=3$

b. If $g(x) = 3 - x^2$ then $g(5) = ?$

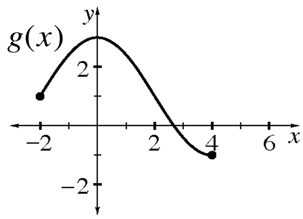
$g(5)=3-(5)^2$
$g(5)=3-25$
$g(5)=-22$

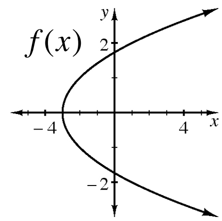
c. If $f(x)=\frac{x+3}{2x-5}$ then $f(2)=?$

$f(2)=\frac{2+3}{2\cdot 2-5}$
$f(2)=\frac{5}{-1}$
$f(2)=-5$

Example 3

A relationship in which each input has only one output is called a **function**.





$g(x)$ is a function; each input (x) has only one output (y).
$g(-2) = 1$, $g(0) = 3$, $g(4) = -1$, and so on.

$f(x)$ is not a function: each input greater than –3 has two y-values associated with it. $f(1) = 2$ and $f(1) = -2$.

Example 4

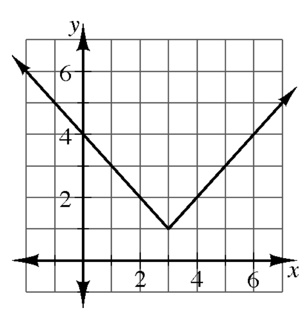
The set of all possible inputs is called the **domain**, while the set of all possible outputs of is called the **range**.

In Example 3 above, the domain of $g(x)$ is $-2 \le x \le 4$, or "all numbers between –2 and 4". The range of $g(x)$ is $-1 \le g(x) \le 3$ or "all numbers between –1 and 3".

The domain of $f(x)$ in Example 3 above is $x \ge -3$ or "any real number greater than or equal to 3," since the graph starts at $x = -3$ and continues forever to the right. Since the graph of $f(x)$ extends in both the positive and negative y (vertical) directions forever, the range is "all real numbers".

Example 5

For the graph at right, since the graph extends forever horizontally in both directions, the domain is "all real numbers". The y-values start at $y = 1$ and increase, so the range is $y \ge 1$ or "all numbers greater or equal to 1".



Problems

Determine the outputs for the following function machines and the given inputs.

1. $x = 2$ → $f(x) = -2x + 4$ → $f(x) = ?$

2. $x = -6$ → $f(x) = |x - 2|$ → $f(x) = ?$

3. $x = 9$ → $f(x) = \sqrt{x} + 1$ → $f(x) = ?$

4. $f(x) = (5 - x)^2$
   $f(8) = ?$

5. $g(x) = x^2 - 5$
   $g(-3) = ?$

6. $f(x) = \frac{2x+7}{x^2-9}$
   $f(3) = ?$

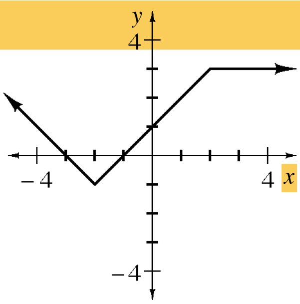
7. $h(x) = 5 - \sqrt{x}$
   $h(9) = ?$

8. $h(x) = \sqrt{5 - x}$
   $h(9) = ?$

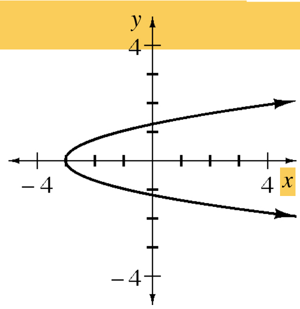
9. $f(x) = -x^2$
   $f(4) = ?$

Determine if each graph below represents a function.

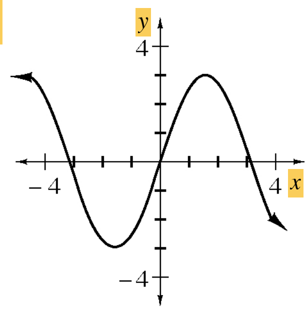
10.



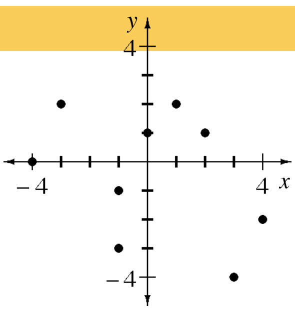
11.



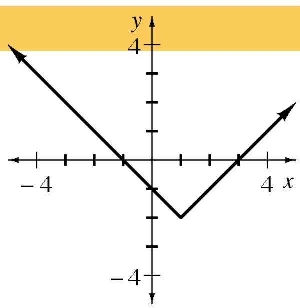
12.



13.



14.



15.