A relationship between the input values (usually $x$ ) and the output values (usually $y$ ) is called a function if for each input value, there is no more than one output value. Functions can be represented with an illustration of a "function (input-output) machine", as shown in Lesson 1.2.3 of the textbook and in the diagram in Example 1 below.
Note: $f(x)=2 x+1$ is equivalent to $y=2 x+1$.
The set of all possible inputs is called the domain, while the set of all possible outputs is called the range.

For additional information about functions, function notation, and domain and range, see the Math Notes box in Lesson 1.2.3.

## Example 1

The inputs of a function are " $x$ "s and the outputs are " $f(x)$ "s. Numbers are input into the function machine labeled $f$ one at a time, and then the function performs the indicated operation on each input to determine its corresponding output. For example, when $x=3$ is put into the function machine $f$ at right, the function multiplies the 3 by 2 and then adds 1 to get the corresponding output, which is 7 . The notation $f(3)=7$ shows that the function named $f$ connects the input 3 with the corresponding output 7 . This also means the point $(3,7)$ lies on the graph of the function.

## inputs


outputs

## Example 2

a. If $f(x)=\sqrt{x-2}$ then $f(11)=? \quad f(11)=\sqrt{11-2}$

$$
f(11)=\sqrt{9}
$$

$$
f(11)=3
$$

b. If $g(x)=3-x^{2}$ then $g(5)=$ ?

$$
\begin{aligned}
& g(5)=3-(5)^{2} \\
& g(5)=3-25 \\
& g(5)=-22
\end{aligned}
$$

c. If $f(x)=\frac{x+3}{2 x-5}$ then $f(2)=$ ?

$$
\begin{aligned}
& f(2)=\frac{2+3}{2 \cdot 2-5} \\
& f(2)=\frac{5}{-1} \\
& f(2)=-5
\end{aligned}
$$

## Example 3

A relationship in which each input has only one output is called a function.

$g(x)$ is a function; each input ( $x$ ) has only one output ( $y$ ).
$g(-2)=1, g(0)=3, g(4)=-1$, and so on.

$f(x)$ is not a function: each input greater than -3 has two $y$-values associated with it. $f(1)=2$ and $f(1)=-2$.

## Example 4

The set of all possible inputs is called the domain, while the set of all possible outputs of is called the range.

In Example 3 above, the domain of $g(x)$ is $-2 \leq x \leq 4$, or "all numbers between -2 and 4 ". The range of $g(x)$ is $-1 \leq g(x) \leq 3$ or "all numbers between -1 and 3 ".

The domain of $f(x)$ in Example 3 above is $x \geq-3$ or "any real number greater than or equal to 3 ," since the graph starts at $x=-3$ and continues forever to the right. Since the graph of $f(x)$ extends in both the positive and negative $y$ (vertical) directions forever, the range is "all real numbers".

## Example 5

For the graph at right, since the graph extends forever horizontally in both directions, the domain is "all real numbers". The $y$-values start at $y=1$ and increase, so the range is $y \geq 1$ or "all numbers greater or equal to 1 ".


## Problems

Determine the outputs for the following function machines and the given inputs.
1.
$x=2$
2.

3.
$x=9$

4.

$$
\begin{aligned}
& f(x)=(5-x)^{2} \\
& f(8)=?
\end{aligned}
$$

5. $g(x)=x^{2}-5$
$g(-3)=$ ?
6. $f(x)=\frac{2 x+7}{x^{2}-9}$
$f(3)=$ ?
7. $h(x)=5-\sqrt{x}$ $h(9)=$ ?
8. $h(x)=\sqrt{5-x}$
$h(9)=$ ?
9. $f(x)=-x^{2}$ $f(4)=$ ?

Determine if each graph below represents a function. Then state its domain and range.
10.

11.

12.

13.

14.

15.


